classmate

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★ Th. ① Let $V(F)$ be a v.s. finite dimensional and $W < V$. Then, $\dim\left(\frac{V}{W}\right) = \dim(V) - \dim(W)$

Solⁿ: If Basis of $V = \{a_1, a_2, \dots, a_m, b_1, \dots b_n\}$
$\dim.$ of $V = m+n$
Basis of ~~dim of~~ $W = \{a_1, a_2, \dots a_m\}$
$\dim. W = m$.
Basis of $V/W = \{b_1+W, b_2+W, \dots, b_n+W\}$
$\therefore \dim.$ of $V/W = n$

$\therefore$ $\text{Dim. } V/W = n+m-n = \text{Dim. } V - \dim W$.

Th. ② Let $V(F)$ be a F.D. V.S. and W_1 & W_2 are two subspaces of V s.t. $V = W_1 \oplus W_2$. If $\{a_1, a_2, \dots a_n\}$ is basis of W_1, then basis of V/W_2 will be $\{a_1+W_2, a_2+W_2, \dots, a_n+W_2\}$

Solⁿ: given basis of $W_1 = \{a_1, a_2, \dots, a_n\}$
$V = W_1 \oplus W_2$
Let Basis of $W_2 = \{b_1, b_2 \dots, b_m\}$ (let)
Then Basis of $V = \{a_1, a_2, \dots, a_n, b_1, b_2, \dots b_m\}$

Then, Basis of $V/W_1 = \{b_1+W_1, b_2+W_1, \dots, b_m+W_1\}$
& Basis of $V/W_2) = \{a_1+W_2, a_2+W_2, \dots + a_n+W_2\}$

$\dim$ of $(V/W_1) = m = \dim W_2$
& $\dim$ of $(V/W_2) = n = \dim. W_1$

Q: Let $V(R) = R^3(R)$ s.t. $W_1 = \{(x,0,0) : x \in R\} < V$
$W_2 = \{(0,y,z) : y, z \in R\} < V$

Solⁿ: $V = W_1 \oplus W_2 = $ ~~$\{(x,y,z)$~~ ; Basis of $V = \{(1,0,0), (0,1,0), (0,0,1)\}$
Basis of $W_1 = \{(1,0,0)\}$; $W_2 = \{(0,1,0), (0,0,1)\}$